

2M/MTH-150 Syllabus-2023

2026

(May-June)

FYUP : 2nd Semester Examination

MATHEMATICS

(Major)

(**Fundamental Mathematics—II**)

(MTH-150)

Marks : 75

Time : 3 hours

*The figures in the margin indicate full marks
for the questions.*

Answer **four** questions, selecting **one** from each Unit.

UNIT—I

1. (a) Transform the equation

$$3x^2 + 8xy + 3y^2 - 2x + 2y - 2 = a^2$$

referred to new axes through $(-1, 1)$

rotated through an angle $\frac{\pi}{4}$.

5

- (b) Prove that the equation

$$2x^2 - 5xy + 3y^2 - 2x + 3y = 0$$

represents two lines and find their point of intersection.

5

- (c) Through what angle must the axes be rotated so that the term of xy may be removed from the equation

$$ax^2 + 2hxy + by^2 = 0 ?$$

4

- (d) Find the diameter of the conic $4x^2 + 6xy - 5y^2 = 1$ conjugate to the diameter $y = 2x$.

4

2. (a) If the two pairs of lines

$$x^2 - 2pxy - y^2 = 0 \text{ and } x^2 - 2qxy - y^2 = 0$$

be such that each pair bisects the angles between the other pair, prove that $pq + 1 = 0$.

5

- (b) Reduce the equation

$$3x^2 - 6xy - 5y^2 - 6x + 22y - 17 = 0$$

to standard form.

7

- (c) Find the equation of the tangent to the conic

$$4x^2 + 3xy + 2y^2 - 3x + 5y + 7 = 0$$

at the point $(1, -2)$.

3

- (d) Determine the conic-section represented by the following equations :

$$1\frac{1}{2} + 1\frac{1}{2} = 3$$

(i) $4x^2 - 20xy + 25y^2 - 7x + 11y - 8 = 0$

(ii) $7x^2 - 2xy + 7y^2 - 16x + 16y - 8 = 0$

UNIT—II

3. (a) Find the equation of a cone whose vertex is $(1, -2, 1)$ and whose guiding curve is $z = 0, 2x^2 + y^2 = 1$.

5

- (b) Find the equations of the two tangent planes to the sphere $x^2 + y^2 + z^2 = 9$ which passes through the line $\{x + y = 6, x - 2z = 3\}$.

5

- (c) Find the equation of the plane passing through the intersection of the planes

$$2x + y - z = 9 \text{ and } 7x - 3y + 4z = 2$$

and through the point $(3, -1, 2)$.

5

- (d) Find the centres and radii of the following spheres :

$$2 + 2 = 4$$

(i) $x^2 + y^2 + z^2 - 6x + 8y - 10z + 1 = 0$

(ii) $2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z + 3 = 0$

4. (a) Find the equation of a plane which is perpendicular to the plane $5x + 3y + 6z + 8 = 0$ and which contains the line of intersection of the plane $x + 2y + 3z - 4 = 0$ and $2x + y - z + 5 = 0$. 6
- (b) Obtain the equation of a sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as a great circle. 5
- (c) Find the enveloping cone of the sphere $x^2 + y^2 + z^2 - 2x + 4z = 1$ with its vertex at $(1, 1, 1)$. 5
- (d) Find the angle between the planes $2x - y + z = 6$ and $x + y + 2z = 3$. 3

UNIT—III

5. (a) Find the domain and range of $f(x, y) = \frac{1}{\sqrt{x+y-1}}$ 4
- (b) Determine whether the function f defined by $f(x, y) = \begin{cases} xy \frac{x^2 - y^2}{x^2 + y^2}, & \text{where } x^2 + y^2 \neq 0 \\ 0, & \text{where } x = y = 0 \end{cases}$ is continuous at $(0, 0)$. 5

- (c) If $f(x, y) = x \cos y + ye^x$, then find $\frac{\partial^2 f}{\partial x^2}, \frac{\partial^2 f}{\partial y \partial x}, \frac{\partial^2 f}{\partial y^2}$ and $\frac{\partial^2 f}{\partial x \partial y}$. 5
- (d) If $u = \sin^{-1} \frac{5xy + 3x^2}{\sqrt{4x + 5y}}$ show that $3 \left(x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} \right) = 5 \tan u$ 5
6. (a) Verify Euler's theorem for $f(x, y, z) = axy + byz + czx$ 4
- (b) Let $f(x, y)$ be a real valued function defined by $f(x, y) = \begin{cases} \frac{x^2 y^2}{x^2 + y^2}, & x^2 + y^2 \neq 0 \\ 0, & x^2 + y^2 = 0 \end{cases}$ Evaluate $f_{xy}(0, 0)$ and $f_{yx}(0, 0)$ using definition. 5
- (c) Show that the double limit exists but repeated limits do not exist of the function $f(x, y) = \begin{cases} x \sin \frac{1}{y} + y \sin \frac{1}{x}, & \text{when } x, y \neq 0 \\ 0, & \text{when } x = y = 0 \end{cases}$ 5

- (d) Find the degree of the homogeneous function

$$f(x, y) = \frac{x^{1/4} + y^{1/4}}{x^{1/5} + y^{1/5}}$$

and show that

$$2x \frac{\partial f}{\partial x} + 2y \frac{\partial f}{\partial y} = f(x, y) \quad 5$$

UNIT—IV

7. (a) Prove that

$$(\vec{a} \times \vec{b}) \cdot (\vec{c} \times \vec{d}) = \begin{vmatrix} \vec{a} \cdot \vec{c} & \vec{a} \cdot \vec{d} \\ \vec{b} \cdot \vec{c} & \vec{b} \cdot \vec{d} \end{vmatrix} \quad 5$$

- (b) If $\vec{u} = 5t^2\hat{i} + t\hat{j} - t^3\hat{k}$ and $\vec{v} = \sin t\hat{i} - \cos t\hat{j}$, then find

(i) $\frac{d}{dt}(\vec{u} \cdot \vec{v})$

(ii) $\frac{d}{dt}(\vec{u} \cdot \vec{u}) \quad 3+2=5$

- (c) Find the directional derivative of the function $f = x^2 - y^2 + 2z^2$ at the point $P(1, 2, 3)$ in the direction of the line PQ , where Q has coordinates $(5, 0, 4)$. 5

- (d) Verify that $\text{curl grad } f = \vec{0}$, where $f = x^2y + 2xy + z^2$. 4

8. (a) Show that

$$\vec{\nabla} \left(\frac{1}{r} \right) = -\frac{\vec{r}}{r^3}$$

where $r = |\vec{r}|$. 5

- (b) Find the unit vector normal to the surface $\phi = 2x^2y + 3yz - 4$ at the point $(1, -1, -2)$. 5

- (c) Determine the divergence and curl of the vector

$$\vec{f} = (x^2 - y^2)\hat{i} + 2xy\hat{j} + (y^2 - xy)\hat{k} \quad 5$$

- (d) If $\vec{a} = t^2\hat{i} - t\hat{j} + (2t+1)\hat{k}$, and $\vec{b} = (2t-3)\hat{i} + \hat{j} - t\hat{k}$, then find

$$\frac{d}{dt} \left(\vec{a} \times \frac{d\vec{b}}{dt} \right) \text{ at } t = 2. \quad 4$$

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